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A Note on Linear Transformation.

By H. P. Manning.

The following was suggested by a method which Professor Cayley has employed in his "Memoir on the Abelian and Theta Functions," in the *American Journal of Mathematics*, Vol. V.

Suppose we have $f(x_1, x_2, \ldots . x_k)$, a polynomial in k variables, and wish to transform it into a function of y's by the substitution

$$x_i = \sum_{j=1}^{j=k} \alpha_{ij} y_j \qquad (i = 1, 2, \ldots . k).$$

Represent it symbolically by a_x^n where

$$a_x = a_1 x_1 + a_2 x_2 + \ldots . + a_k x_k,$$

and let the transform be A_y^n, where

$$A_y = A_1 y_1 + A_2 y_2 + \ldots . + A_k y_k.$$

If we let δ_i denote the polar operation, namely,

$$\delta_i = \alpha_{1i} \frac{\partial}{\partial x_1} + \alpha_{2i} \frac{\partial}{\partial x_2} + \ldots . + \alpha_{ki} \frac{\partial}{\partial x_k},$$

then

$$A_i = \delta_i a_x,$$
$$A_i A_j = \delta_i a_x . \delta_j a_x = \tfrac{1}{2} \delta_i \delta_j a_x^2,$$

and in general,

$$A_i^a A_j^b \ldots . = \frac{1}{(a + b + \ldots .)!} \delta_i^a \delta_j^b \ldots . a_x^{a+b+\ldots .}.$$

Hence A_y^n may be written

$$\frac{1}{n!} (y_1 \delta_1 + y_2 \delta_2 + \ldots . + y_k \delta_k)^n a_x^n,$$

that is, f becomes

$$\frac{1}{n\,!}(y_1\delta_1 + y_2\delta_2 + \ldots\ldots + y_k\delta_k)^n f.$$

We can write down at once a formula for the coefficient of any term:

$$\text{coeff. of } y_i^a y_j^b \ldots\ldots = \frac{1}{a\,!\;b\,!\;\ldots\ldots}\,\delta_i^a\delta_j^b\ldots\ldots f \qquad (a + b + \ldots\ldots = n)\cdot$$

In each case there are n polar operations, and these will always introduce the same numerical factor, $n\,!$

The n polar operations remove all the x's and replace them by α's. Moreover, $\delta_i^n f$ is equivalent to f with all the x's replaced by α_{1i}, $\alpha_{2i}\ldots\ldots$, and we may in any particular case reduce the number of polar operations actually to be performed by a number equal to the highest exponent in the term. For example, to obtain $\delta_1^3\delta_5^{n-3}f$ we have only to perform the operation

$$\left(\alpha_{11}\frac{\partial}{\partial\alpha_{15}} + \alpha_{21}\frac{\partial}{\partial\alpha_{25}} + \ldots\ldots\right)^3 f(\alpha_{15}, \alpha_{25}, \ldots\ldots).$$

Example: Suppose we wish to transform the cubic

$$f = ax^3 + by^3 + cz^3 - 6lxyz$$

by the substitution

$$\begin{aligned} x &= \alpha_1 X + \alpha_2 Y + \alpha_3 Z, \\ y &= \beta_1 X + \beta_2 Y + \beta_3 Z, \\ z &= \gamma_1 X + \gamma_2 Y + \gamma_3 Z, \end{aligned}$$

we have

$$\begin{aligned} \delta_i &= \alpha_i\frac{\partial}{\partial x} + \beta_i\frac{\partial}{\partial y} + \gamma_i\frac{\partial}{\partial z}, \\ \delta_i f &= 3\,(a\alpha_i x^2 + b\beta_i y^2 + c\gamma_i z^2 - 2l\alpha_i yz - 2l\beta_i zx - 2l\gamma_i xy), \\ \delta_i\delta_j f &= 6\,(a\alpha_i\alpha_j x + \ldots\ldots - l\alpha_i\beta_j z - \ldots\ldots); \\ \therefore\ f &= (a\alpha_1^3 + b\beta_1^3 + c\gamma_1^3 - 6l\alpha_1\beta_1\gamma_1)\,X^3 + \ldots\ldots \\ &\quad + 3\,(a\alpha_1^2\alpha_2 + \ldots\ldots - 2l\beta_1\gamma_1\alpha_2 - \ldots\ldots)\,X^2 Y + \ldots\ldots \\ &\quad + 6\,(a\alpha_1\alpha_2\alpha_3 + \ldots\ldots - l\alpha_1\beta_2\gamma_3 - \ldots\ldots)\,XYZ. \end{aligned}$$